# RELATIVITY PARAMETERS DETERMINED FROM LUNAR LASER RANGING


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ABSTRACT

Laser ranges to retroreflectors placed on the Moon by lunar space missions have been collected over the past 24 years. A comprehensive set of parameters relating to the Earth and Moon can be estimated from these data. This paper presents current lunar laser ranging results for five relativity parameters: the Principle of Equivalence, geodetic precession, the Parametrized Post-Newtonian parameters $\beta$ and $\gamma$, and a possible time rate of change of the gravitational constant $G$. Neither a violation of general relativity nor a change in $G$ is found.


## 1. Introduction

Beginning in 1969, lunar space missions placed four corner-cube retroreflectors on the lunar surface. A "range" is the elapsed round-trip travel time of a pulse of light between a terrestrial observatory and a lunar retroreflector. As of January, 1994, more than 8400 lunar laser ranging (1,1,1 /.) normal points have been c.ollm.tee] from three observatories, with accuracies over the past five years being 2-3 **cm**, Among parameters estimated arc five related to relativity.

## 2. The Principle of Equivalence

A consequence of the Principle of Equivalence is that the gravitational mass $M_G$ of any object is identical to its inertial mass $M_I$. (The Weak Equivalence Principle (WEP) holds if independent of composition; the Strong Equivalence Principle (SEP) holds if true for gravitational self-energy.) A failure of this principle would lead to the Nordtvedt effect, in which the geocentric lunar orbit is displaced in the direction of the Sun, and would exhibit a signature in the LLR data ([1], [2], [3]).

The quantity measured by LLR is $(M_G/M_I)_{Earth} - (M_G/M_I)_{Moon}$. A correction for solar radiation pressure [4] is applied to the estimated value. With this correction,

$$\frac{M_G}{M_I}\Big|_{Earth} - \frac{M_G}{M_I}\Big|_{Moon} = (3 \pm 5) \times 10^{-13}$$

The dependence of the ratio $M_G/M_I$ on the Strong Equivalence Principle can be expressed as $M_G/M_I = 1 + \eta(U_G/Mc^2)$ ([1], [2], [3]), where $\eta$ is a dimensionless parameter and $U_G$ is the gravitational self-energy of the celestial body. For the Earth and Moon, $(U_G/Mc^2)_{Earth} - (U_G/Mc^2)_{Moon} = -4.45 \times 10^{-10}$. The LLR result for $\eta$ is

$$\eta = -0.0043 \pm 0.0051 \quad \text{if WEP uncertainty is included}$$
$$-0.0007 \pm 0.0010 \quad \text{if WEP is assumed}$$

where the results of Su *et al.* [5] arc used for the WEP.

## 3. Geodetic Precession

General relativity predicts that the lunar node, longitude of perigee, and mean longitude should precess at 19.2 milliarcseconds/year. This rate $P_g$ is implicit in the equations of motion used to model the Moon and planets. The LLR result is

$$\frac{\Delta P_g}{P_g} = 0.3\% \pm 0.7\%$$

## 4. Parametrized Post-Newtonian $\beta$ and $\gamma$

Two Parametrized Post-Newtonian (PPN) parameters can be estimated from LLR data: $\beta$, measuring non-linearity of superposition, and $\gamma$, measuring space curvature produced by unit mass. Both $\beta$ and $\gamma$ appear in the equations of motion; $\gamma$ also appears in the light-time equation. Using only these formulations,

$$|\beta - 1| \leq 0.005, \quad |\gamma - 1| \leq 0.005, \quad |\beta + \gamma - 2| \leq 0.003$$

Nordtvedt ([1], [2], [3]) showed that $\beta = (\eta + \gamma + 3)/4$. Using $\eta$ from the Equivalence Principle above and $|\gamma - 1| \leq 0.002$ from interplanetary ranging [6],

$$\begin{aligned} |\beta - 1| &< 0.0014 \quad \text{if WEP uncertainty is included} \\ &< 0.0006 \quad \text{if WEP is assumed} \end{aligned}$$

## 5. Rate of Change of the Gravitation Constant

The present LLR data span gives sensitivity to any time rate of change in the gravitation constant $G$, primarily through the solar perturbation on the lunar orbit. The change in G is expressed as $\dot{G}/G$. The result from LLR is

$$\frac{\dot{G}}{G} = (0.1 \pm 0.9) \times 10^{-11} \text{/year}$$

## 6. summary

No significant change from general relativity is found, and no change in $G$ is evident. A more extensive discussion of this work is in [7].


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